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Solution of the Implicit Colebrook Equation for Flow Friction Using Excel

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Abstract
Empirical Colebrook equation implicit in unknown flow friction factor (λ) is an accepted standard for calculation of hydraulic resistance in hydraulically smooth and rough pipes. The Colebrook equation gives friction factor (λ) implicitly as a function of the Reynolds number (Re) and relative roughness (ε/D) of inner pipe surface; i.e. λ_0=f(λ_0, Re, ε/D). The paper presents a problem that requires iterative methods for the solution. In particular, the implicit method used for calculating the friction factor λ_0 is an application of fixed-point iterations. The type of problem discussed in this "in the classroom paper" is commonly encountered in fluid dynamics, and this paper provides readers with the tools necessary to solve similar problems. Students' task is to solve the equation using Excel where the procedure for that is explained in this "in the classroom" paper. Also, up to date numerous explicit approximations of the Colebrook equation are available where as an additional task for students can be evaluation of the error introduced by these explicit approximations $\lambda \approx$f(Re, ε/D) compared with the iterative solution of implicit equation which can be treated as accurate.

Keywords
Colebrook equation, Hydraulic friction, Turbulence, Pipes, Flow

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Introduction

Hydraulic resistance depends on flow rate. Similar situation is with electrical resistance when a diode is in circuit [1]. To be more complex, widely used empirical Colebrook equation (1) is iterative i.e. implicit in fluid flow friction factor because the unknown friction factor (λ) appears on both sides of the equation [2].

$$\frac{1}{\sqrt{\lambda_0}} = -2 \cdot \log_{10}\left(\frac{2.51}{\mathrm{Re} \cdot \sqrt{\lambda_0}} + \frac{\varepsilon}{3.71 \cdot D}\right) \tag{1}$$

In Colebrook's equation, λ is Darcy flow friction factor (dimensionless), Re is Reynolds number (dimensionless) and ε/D is relative roughness of inner pipe surface (dimensionless). Practical domain of the Reynolds number (Re) is between 4000 and 10^8 while for the relative roughness (ε/D) is up to 0.05. Index 0 here denotes the values of friction factor (λ) calculated using the implicit Colebrook equation through iterative procedure, i.e. it denotes the solution conditionally accepted as accurate, or let's say the most accurate compared with other approaches such as use of explicit approximate formulas.

The Colebrook equation is based on joint experiment which Colebrook as PhD student conducted with his professor White [3]. Later Rouse followed by Moody made flow friction diagram based on these results [4, 5].

The Colebrook equation is valuable for determination of hydraulic resistances for turbulent regime in smooth and rough pipes but it is not valid for laminar regime. It describes a monotonic change in the friction factor in commercial pipes from smooth to fully rough. This equation has become the accepted standard of accuracy for calculation of hydraulic friction factor despite the fact that many new experiments have disputed its accuracy [6].

The empirical and implicit Colebrook equation cannot be rearranged to derive and calculate friction factor (λ) directly in one step [7]. The most accurate procedure to calculate this unknown friction factor (λ) is through iterative procedure [8]; $\lambda_0=f(\lambda_0, \mathrm{Re}, \varepsilon/D)$. This can be accomplished relative easily in spreadsheet environment and the detailed procedure is explained in this "in the classroom" paper. In addition to the iterative procedure, many explicit approximations are available; $\lambda \approx f(\mathrm{Re}, \varepsilon/D)$, but they introduce certain error [9] which can be predicted in advance and which is not distributed uniformly through the practical domain of the Reynolds number (Re) and the relative roughness (ε/D) [10]. An additional task for students is evaluation of this relative error caused by using of approximations compared with the iterative solution which can be treated as accurate [11].

In summary, the main students' tasks are:

1. To calculate flow friction (λ_0) in Excel using implicit Colebrook's equation, and

2. To calculate flow friction (λ) in Excel using explicit approximations of Colebrook's equation and to evaluate relative error. In addition diagrams that represent distribution of error can be drawn in Excel.

3. Additional tasks: Lambert W-function, networks of pipes with loops, MATLAB (Genetic Algorithms – GA and Artificial Neural Networks - ANN), Excel fitting tool, etc.

This "in the classroom" paper contains Excel file as Electronic Annex.

1. Iterative solution in Excel using implicit Colebrook equation

To solve the implicit Colebrook equation, one must start by somehow estimating the value of the friction factor (λ_0) on the right side of the equation, to calculate the new λ_0 on the left, enter the new value of λ_0 back on the right side, and continue this process until there is a balance on both sides of the equation within an arbitrary small difference without causing endless computations.

The Colebrook equation can be expressed as (2):

$$\frac{1}{\sqrt{\lambda_0}} = -2\cdot\log_{10}\left(\frac{2.51}{\mathrm{Re}\cdot\sqrt{\lambda_0}} + \frac{\varepsilon}{3.71\cdot D}\right) = -2\cdot\log_{10}\left(\frac{a}{\sqrt{\lambda_0}} + B\right) = -2\cdot\log_{10}(A+B) \qquad (2)$$

Under the logarithm, the term A represents partially turbulent flow through hydraulically smooth pipes proposed by Prandtl while the second term, B, represents turbulent flow through hydraulically rough pipes proposed by von Karman. As can be seen from Figure 1, one pipe can be hydraulically smooth or rough not only depending on the state of its inner roughness but also on the state of boundary sub-layer of fluid in motion near the inner wall surface of the pipe [12].

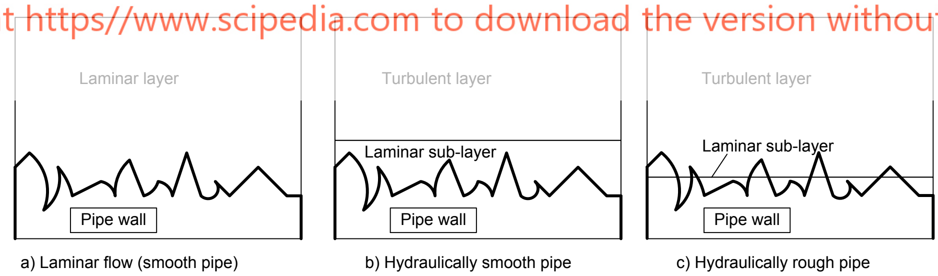

Figure 1: Different hydraulic regimes of flow in one pipe

Using Prandtl's and von Karman's equations separately the sharp change in values of friction factor between smooth and rough regime will occur. On the other hand Colebrook and White during their experiments did not detect this sharp change. According to them the transition from the hydraulically smooth regime of turbulence to the fully rough is smooth as can be seen from Figure 2. Note that log(A)+log(B)≠log(A+B), where the separate use of log(A) and log(B) produce two lines in related diagrams with sharp intersection while log(A+B) produces one smooth line.

So, Colebrook's equation has virtually two parts, smooth Prandtl (A in eq. 2) and rough von Karman (B in eq. 2) with smooth transition between them. Only smooth Prandtl's part is implicit in unknown flow friction factor (λ_0). Knowing that only the first smooth part is equal to zero in the first iteration (A=0; Re–›∞), while the second part has value different than zero (B≠0), the arbitrary estimation of the value of the flow friction factor (λ_0) in the first iteration can be avoided where the initial value in the first iteration is from $\frac{1}{\sqrt{\lambda_0}} = -2 \cdot \log_{10}(B)$.

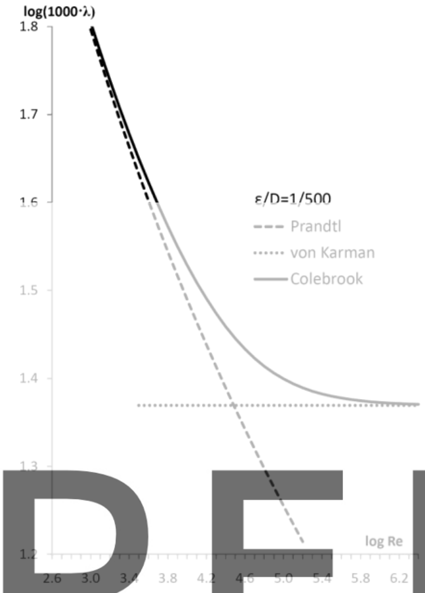

Figure 2: Findings of Colebrook and White shows smooth transitions from smooth to rough turbulent flow

To implement this procedure in Excel in order to solve the implicit Colebrook-White equation the 'Office button' at the upper left corner of the screen need to be pressed (Figure 3) where in 'Excel options', 'Formulas' needs to be selected (this procedure can be slightly different in some version of Excel). As shown in Figure 4, in the window 'Formulas', box 'Enable iterative calculation', need to be ticked and desired number of iteration need to be chosen (max. allowed is 32767).

Excel-code for the implicit Colebrook equation is (result will appear in C1);

=-2*LOG10(((1/3.71)*B1)+((2.51/A1)*C1)) where B1 is cell with the relative roughness (ε/D), A1 cell with the Reynolds number (Re) and C1 is iterative reference.

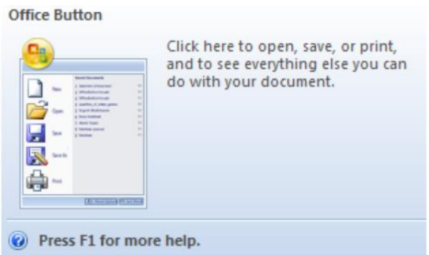

Figure 3: Office button in Excel

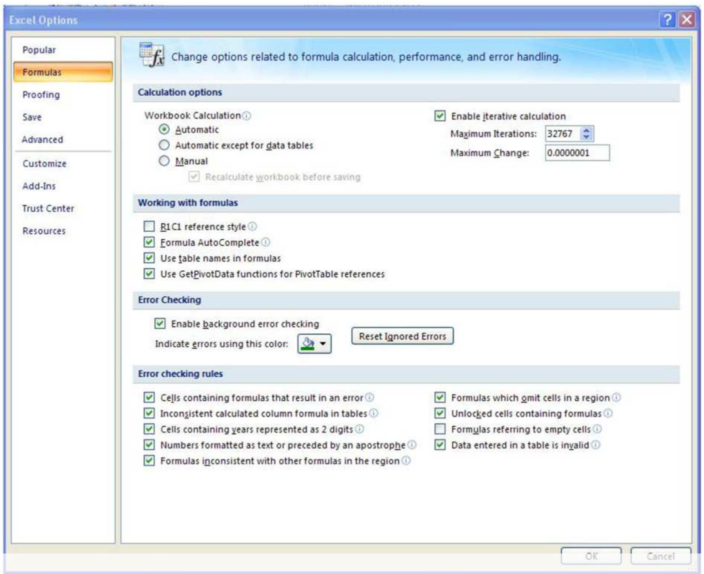

Figure 4: Settings for iterative calculation in Excel

For example:

-for Re=10^4 and ε/D=10^{-6}, λ_0=0.0308844939;

-for Re=$5.8\cdot10^6$ and ε/D=$3\cdot10^{-3}$, λ_0=0.0261693581;

-for Re=$3\cdot10^7$ and ε/D=$4.3\cdot10^{-4}$, λ_0=0.0161582229;

-for Re=$6\cdot10^4$ and ε/D=$2\cdot10^{-4}$, λ_0=0.0208369171;

-for Re=$4\cdot10^5$ and ε/D=0.03, λ_0=0.0571868356; etc.

Results are obtained from Excel file attached to this "in the classroom paper" as Electronic Annex.

2. Explicit approximations of Colebrook's equation

Numerous of explicit approximations of Colebrook's equation exist [13-35]. They introduce certain error which can be estimated in advance. The error is not distributed uniformly through the domain of the Reynolds number (Re) and the relative roughness (ε/D).

Students' task is to find few approximations of the Colebrook equation in available literature [10, 11, 36-41] and to estimate their relative error compared with the accurate iterative solution of the original Colebrook's equation. For this purpose friction factor calculated using approximations can be noted as λ while from the original implicit equation as λ_0. In that way relative percentage error can be calculated as $\delta_{\%}=[(\lambda-\lambda_0)/\lambda_0]\cdot100\%$. Also the whole domain of applicability of the Colebrook equation can be covered with mesh where in nodes the relative error can be calculated. In that way diagram of error can be constructed. Good resolution for that should be achieved with at least 500 mesh nodes over the whole practical domain of the Reynolds number (Re) and the relative roughness (ε/D).

Following calculation of the error performed in the Excel file from Annex of this "in the classroom" paper, students need to prepare an additional Excel file in which error analysis will be performed in order to construct diagram of error distribution. In this file mesh which will allow construction of diagram of error over the domain of the Reynolds number (Re) and the relative roughness (ε/D) is formed.

As an illustrative example, Brkić approximation [14] is examined (3). First part in Eq. 3 is with the original values of coefficients while the second is altered using genetic algorithms [36, 37] in order to decrease maximal relative error ($\delta_{\%}$). Distribution of the relative error of Brkić approximation before and after genetic optimisation [36] can be seen in Figure 5. Maximal relative error before optimisation [14] is about 2.2% and after [36] about 1.29%. Note that the optimisation that is performed to cut maximal error over the domain [36, 37]. Such approach in many cases can cause increase of the relative error in certain points of the domain.

Related this task, students need to produce similar diagrams such as 3D as in Figure 5, but also 2D as well as other appropriate types.

$$\left.\begin{array}{l} \frac{1}{\sqrt{\lambda}} \approx -2 \cdot \log_{10}\left(\frac{2.18 \cdot a_1}{\mathrm{Re}} + \frac{1}{3.71} \cdot \frac{\varepsilon}{D}\right) \\ a_1 \approx \ln \frac{\mathrm{Re}}{1.816 \cdot \ln\left(\frac{1.1 \cdot \mathrm{Re}}{\ln(1+1.1 \cdot \mathrm{Re})}\right)} \end{array}\right\} \rightarrow \left\{\begin{array}{l} \frac{1}{\sqrt{\lambda}} \approx -2.013 \cdot \log_{10}\left(\frac{2.261 \cdot A_1}{\mathrm{Re}} + \frac{1}{3.71} \cdot \frac{\varepsilon}{D}\right) \\ A_1 \approx \ln \frac{\mathrm{Re}}{2.479 \cdot \ln\left(\frac{1.1 \cdot \mathrm{Re}}{\ln(1+1.1 \cdot \mathrm{Re})}\right)} \end{array}\right\} \quad (3)$$

For example for $\mathrm{Re}=7\cdot10^4$ and for $\varepsilon/D=10^{-4}$, $\lambda_0=0.019832705$ from iterative procedure, $\lambda=0.019942264$ according to Brkić approximation (3) before with $\delta_{\%}=0.55\%$ and $\lambda=0.019679583$ after genetic approximation with $\delta_{\%}=0.77\%$.

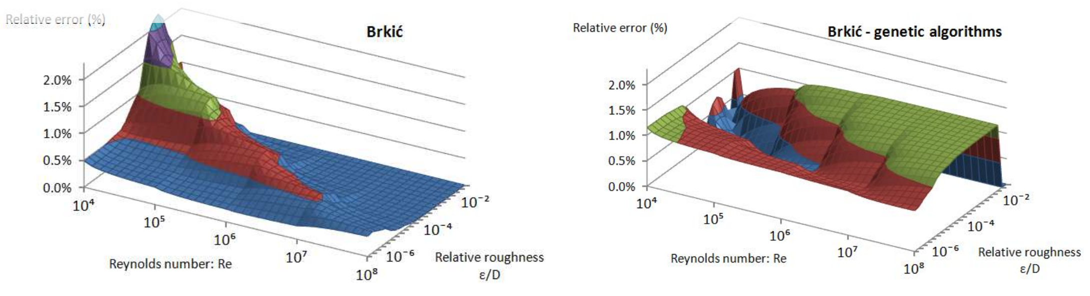

Figure 5: Distribution of error of Brkić approximation before and after genetic optimisation

This "in the classroom" paper is supplied with Excel file which contains certain number of approximations. The file is set also for iterative calculation and hence the error introduced by selected approximation can also be calculated. Using that pattern, students can code in Excel additional approximations found in literature (here as example is shown approximation by Brkić; Eq. 3). Also, as inverse task, already Excelcoded approximation can be extracted from the file and can be compared with the form from original sources [36]. Excel-code for Brkić approximation before optimisation (3) is =-2*LOG10((B1/3.71)+(2.18*E1/A1)) where B1 is cell with the relative roughness (ε/D), A1 cell with the Reynolds number (Re) and E1 auxiliary term =LN(A1/(1.816*LN(1.1*A1/LN(1+1.1*A1)))).

3. Additional tasks

Colebrook equation can be expressed in explicit form through Lambert W-function [42-47]. Further to evaluate friction factor (λ) instead of specific approximations developed for Colebrook's equation, general approximations for the Lambert W-function can be used [48-51]. Students need to find in available literature these approximations of the Lambert W-function, to find available forms of Colebrook's equation expressed through the Lambert W-function [42-45, 52-55] and to implement related calculation in Excel. Students should note that some expressions of Colebrook's equation through the Lambert W-function contain exponential form which makes for some combination of the Reynolds number (Re) and the relative roughness (ε/D) calculation impossible due to limited capability of registers of computer to accommodate extremely large or small numbers [54, 55].

As additional task, student can repeat all activities in e.g. MATLAB or similar software packages. Colebrook equation also can be simulated with Artificial Neural Networks – ANN and such task can be also performed in MATLAB [56].

Excel contains fitting tools [57] which can be used for optimisation of approximations similarly as mentioned optimisation through genetic algorithm which is performed in MATLAB [36, 37]. This activity can be used as a task for advance students.

Further, students can use Excel for more capable task such as calculation of water distribution networks (both tree- and loop-like) where multiple simultaneous calculation of friction factor is needed [58-70].

Conclusion

Colebrook's equation suffers from being implicit in unknown flow friction factor (λ), but on the other hand this equation is relatively simple which makes it ideal for students to train implementation of iterative procedures in spreadsheet environment, to increase their capability to make diagrams, to perform error analysis, etc. All activities can be performed in spreadsheet environment but also in MATLAB or similar software packages specialised for calculation.

The tasks described in this "in the classroom" paper are in the first place for students of hydraulics, petroleum engineering and water resources [71] but also for students of all engineering branches where fluid flow can occur [72-74], including fuel cells [75, 76].

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